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journal homepage: www.elsevier.com/locate/ijsolstrTransverse crack onset and growth in cross-ply $[0/90]_s$ laminates under tension. Application of a coupled stress and energy criterion

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ABSTRACT

Crack initiation in the inner ply of symmetric $[0/90]_s$ laminates under tension is studied applying the coupled stress and energy criterion proposed by Leguillon (Eur. J. Mech. A/Solids 21, 2002) in the framework of the finite fracture mechanics. This criterion assumes that a crack of a finite extension appears abruptly when the stress criterion is fulfilled and this crack onset is energetically admissible.

The stress state is calculated by applying the laminate theory to the undamaged laminate which provides explicit expressions to be used in the stress criterion. Assuming generalised plane strain, the magnitude involved in the (incremental) energy criterion is evaluated numerically by means of a boundary element code and a dimensional analysis. The two criteria lead to a theoretical model providing explicit expressions of the critical parameters for the crack onset, which depend only on the computational results through a scalar value.

This model predicts that the crack grows unstably after the onset in the inner ply and is arrested close to the interface between the plies for a length larger than a certain threshold length independent of the fracture and strength properties. This threshold length depends only on the elastic and geometric properties of the laminate. The model also predicts the existence of a size effect which agrees with the experimental results found in the literature. In addition, the model provides a physical interpretation of this size effect.

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1. Introduction

Fibre reinforced composites are getting very common in structural applications where a high strength-to-weight ratio is required, lightweight being a key aspect of the design, as in aerospace applications. However, predictions of initiation and propagation of damage in composites are still not sufficiently accurate and reliable, which leads to high safety coefficients in design. Consequently, it is necessary to generate more knowledge about mechanisms of damage and failure in composites, see e.g. París (2001).

Cracks appearing and growing in the transverse inner-ply of cross-ply laminates $[0_m/90_n]_s$ represent a classical problem, which has been studied for a long time. Whereas these cracks usually do not significantly reduce the global stiffness of the structure, they are the source of a more dangerous failure mechanism. Basic steps of this mechanism are well known, see reviews by Nairn (2000) and Berthelot (2003). First, above a critical strain level, some cracks appear perpendicular to the load in the inner ply, see e.g. Varna

et al. (2001). Then, some interface cracks appear when transverse cracks reach the interface (or before it) between the inner and outer plies. Finally, coalescence of interface cracks occurs leading to macroscopic delaminations.

The present work focuses on the study of the first transverse crack initiation. Actually, it is also applicable to the sufficiently distant transverse cracks which appear almost simultaneously with the first one. Crack initiation due to unusually large flaws is not taken into account in this work. The problem of the first transverse crack initiation has been studied for decades. The experimental tests carried out by Garrett and Bailey (1977), Parvizi et al. (1978) and Bailey and Parvizi (1981) showed a size effect of the inner-ply thickness on the critical applied strain originating the first transverse crack onset. These experimental results showed that the first failure of the transverse ply is not exclusively either stress-dependent or strain-dependent, which is in the basis of the majority of failure criteria. As a consequence, further theoretical models have been developed dealing with the problem of explaining the size effect found in experiments. Three models stand out among others:

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- Incremental energy criterion model: It was initially proposed by Aveston et al. (1971) in the context of micromechanics of unidirectional fibre composites. Subsequently, it was applied to this problem by Garrett and Bailey (1977). The model is based on assuming that a transverse crack spanning the entire inner-ply thickness appears abruptly. This occurs when the energy released at the crack onset is enough to cover the energy dissipated during the onset. Garrett and Bailey (1977) approximated the released energy by assuming oversimplifying hypotheses about the stresses after the transverse cracking, this fact leading to a quite poor approximation. Hashin (1996) applied this model, improving significantly the approximation, and obtained an analytical expression for the transverse crack density as a function of the external load. He assumed that stresses after the crack onset do not vary within each ply along the direction perpendicular to the ply interface. Employing this assumption, he implemented a variational approach to calculate the released energy. In both approaches, i.e. due to Garrett and Bailey (1977) and Hashin (1996), the physical interpretation of the size effect provided by the model is the same: the released energy varies with the square of the inner-ply thickness, whereas the energy dissipated is linear with this thickness. This cause of size effect will be discussed in depth in Section 6.1.
- Dvořák's model: It was proposed initially by Dvorak and Laws (1986, 1987), see also Dvorak (2013). The key hypothesis is that a certain “damaged zone” or “non-Griffith crack” grows stably when load is increased up to reaching a critical length. Then, the behaviour of the crack becomes governed by the Griffith criterion causing its unstable growth. The critical length is assumed to be material-dependent. According to this assumption and depending on the inner-ply thickness, the “damaged zone” may reach the interface before having reached the critical length. Two possible scenarios lead to two very different behaviours. For inner plies sufficiently thick in comparison with the critical length, the crack growth is essentially driven by the unstable growth along the direction perpendicular to the interface. On the contrary, for thin inner plies, the damaged zone reaches the interface before having the critical length and subsequently it grows along the direction parallel to the fibre in the inner ply. After reaching the critical length along this direction, the crack grows unstably. As a consequence, for thin laminates, the unstable growth is governed by the behaviour along the fibre direction in the inner ply. van der Meer and Dávila (2013) showed that a different behaviour for thick and thin laminates is also predicted when a circular transverse defect is introduced and the growth is simulated by prescribing a cohesive zone model at the surface containing the defect. Therefore, whereas for thick plies the critical strain predicted by this model is independent of the inner-ply thickness, a strong size effect is found for thin plies, which agrees with experiments. Following the work by Dvořák, this model has been widely analysed and extended by other authors, see e.g. Maimí et al. (2011). Some of them assume the preexistence of a crack of a fixed material-dependent length instead of the original Dvořák's hypothesis of “non-Griffith crack” below a critical length. However, both hypotheses lead to equivalent analyses and results. Focusing on the design, a new matrix failure criterion taking into account this model was proposed by Camanho et al. (2006).
- Statistical model: It is based on the well-known Weibull's theory. The key assumption is the pre-existence of defects, e.g. microcracks, flaws or damaged zones with lengths distributed following a Weibull-like law in the whole inner-ply volume, see e.g. Li and Wisnom (1997) and Wisnom (2000). The distribution is defined by two parameters which are assumed to be material-dependent. Since the defects density is statistically

distributed, increasing the volume of material makes more likely to find a larger defect. As a consequence, critical strain for which the first defect becomes a crack decreases with volume. Thus, when the other laminate lengths are fixed, the critical strain is predicted to decrease with the inner-ply thickness as observed in experiments.

Inspired by the experimental results due to Parvizi et al. (1978) among others, Leguillon (2002) proposed, in the framework of the finite fracture mechanics (FFM) introduced by Hashin (1996), a coupled failure criterion based on the combination of a stress criterion and an energy criterion to predict an abrupt onset of a finite-extension crack, see also Cornetti et al. (2006) and Taylor (2007). He explained qualitatively the Parvizi's observations by means of this criterion. Over the last decade this FFM approach has been successfully used to predict critical loads in very different problems covering several scales, see a review by Hebel et al. (2010). The key idea, hereinafter referred to as Leguillon's hypothesis, is very simple: whereas stress and energy criteria applied individually represents two necessary conditions to allow a crack onset, the coupling of both criteria defines a necessary and sufficient condition for it.

The work presented here aims to revisit the problem from the new point of view provided by the FFM and the coupled criterion. Following the generalised plane strain approach widely used to analyse this problem, see e.g. McCartney (1998), a novel 2D theoretical model is developed here following the methodology introduced by Mantić (2009). This approach allows a “semianalytical” expression to be obtained in order to predict the critical strain leading to the first transverse crack onset. This expression is analytical except for a scalar value which is obtained by the computational analysis of the problem. In this case, Boundary Element Method (BEM) code developed by Blázquez et al. (2008, 2009) is used to compute this value. Additionally, this new expression depends only on the well-known laminate properties as geometry, elastic constants, transverse fracture toughness and strength.

The stress criterion is presented as a direct and simple application of the laminate theory in Section 2. After a dimensional analysis of the energy criterion and some other considerations, a final expression of this criterion is presented in Section 3. Additionally, results related to the Energy Release Rate (ERR) of the transverse crack computed by the BEM, suitable for this kind of problem, are presented in this section. A combination of both criteria is introduced in Section 4. Critical applied strain leading to the onset of a transverse crack and the length of this crack at its onset predicted by the model are presented in Sections 4 and 5. Finally, a size effect predicted by the application of the present approach is described in Section 6, and compared with the experimental evidences from the bibliography in Section 7.

2. Stress criterion

The stress criterion is applied to the uncracked cross-ply laminate shown in Fig. 1(a). The laminates under study here are symmetric and composed by an inner transverse-ply with a thickness $2t_{90}$ and two outer and longitudinal plies with an individual thickness t_0 . The two other dimensions of the rectangular specimens under study are $W \times H$. Let (x, y, z) be a suitably defined cartesian coordinate system, the y -axis being coincident with the outer-ply fibre axis and with the load direction, the z -axis being coincident with the inner-ply fibre axis and the x -axis being perpendicular to the interface between the inner and outer ply.

All the plies are of the same orthotropic elastic material (Table 1) and are assumed to be perfectly bonded along their interface. Let E_{11} and E_{22} denote respectively the longitudinal and transverse Young's moduli ($E_{11} > E_{22} = E_{33}$), $\nu_{12} = \nu_{13}$ and ν_{23} the Poisson's

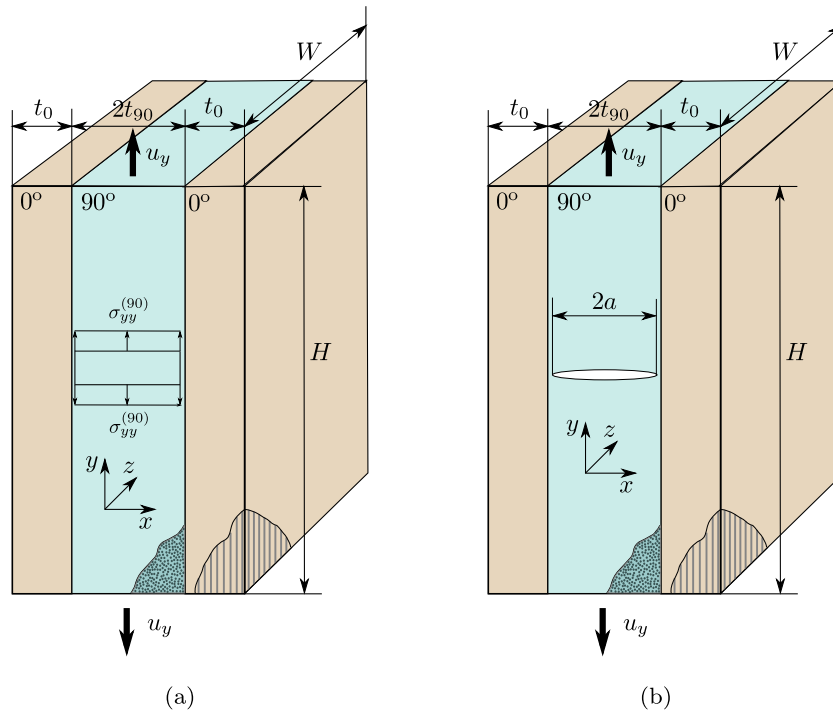


Fig. 1. Schematic representations of the (a) undamaged and (b) cracked laminates.

Table 1
Properties of the two materials used.

Composite	E_{11} (GPa)	E_{22} (GPa)	ν_{12}	ν_{23}	G_{12} (GPa)	G_{23} (GPa)
Glass/epoxy used by Parvizi et al. (1978)	42	14	0.278	0.4	5.83	5
Carbon/epoxy used by París et al. (2010a,b)	141.3	9.58	0.3	0.32	5	3.5

ratios, and $G_{12} = G_{13}$ and G_{23} the transverse shear moduli, the other elastic parameters being defined by these properties. A uniform longitudinal strain ε_{yy} is applied along the y -axis. Geometry, loads and material properties allow us to assume a generalised plane strain state in the xy -plane in the sense used by Blázquez et al. (2006).

The tensile stress criterion applied here assumes the existence of a critical value σ_c as the tension required to originate fracture at a certain plane of the lamina. This value depends on the ply orientation with respect to the load direction because of the material microstructure of the ply, given by approximately parallel long fibres embedded in matrix. In the present problem a crack perpendicular to the load direction is assumed to appear in the 90° lamina. Thus, the value of σ_c can be identified with the transverse strength under tension of the lamina Y_t . According to this criterion, the crack can appear at those points (x, y, z) where the value σ_{yy} of the tension normal to the plane of the crack verifies

$$\sigma_{yy}^{(90)}(x, y, z) \geq Y_t. \quad (1)$$

Note that the sign in the above condition is “exceeding” (\geq) instead of “reaching” ($=$) because the stress criterion is only a necessary condition. Therefore, the critical value for stresses could be exceeded without leading to a crack onset. An analytic solution for stresses is possible under the assumption of the laminate theory, see e.g. Jones (1999). Assuming that a uniform longitudinal strain ε_{yy} is applied, the value of $\sigma_{yy}^{(90)}$ is uniform along the thickness and width of the 90° lamina, $\sigma_{yy}^{(90)} = \tilde{E}_{22}\varepsilon_{yy}$, and the stress criterion leads to a very simple expression

$$\tilde{E}_{22}\varepsilon_{yy} \geq Y_t, \quad (2)$$

where \tilde{E}_{22} is the apparent Young’s modulus of the inner 90° ply in the y -direction. This value can easily be expressed as a function of the laminate properties by the laminate theory as demonstrated by García (2014),

$$\frac{E_{22}}{\tilde{E}_{22}} = \frac{1 - \nu_{12}\nu_{21}}{1 - \nu_{12}\nu_{21} \frac{1 + \frac{t_0}{t_{90}}}{1 + \frac{E_{22}t_0}{E_{11}t_{90}}}}. \quad (3)$$

Fig. 2 shows the value of this ratio as a function of the ratio of the outer to the inner ply thickness t_0/t_{90} for the materials in Table 1.

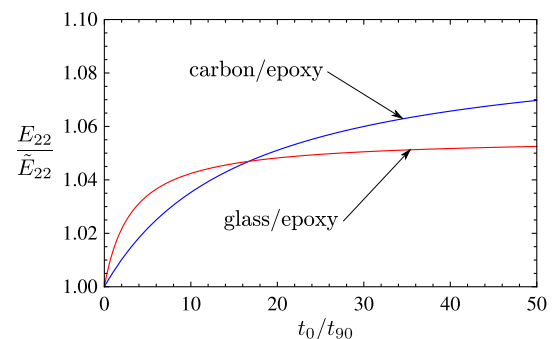


Fig. 2. Ratio of the true (material) to the apparent transverse Young moduli in the inner ply as a function of the ratio of the outer to the inner ply thickness. These values have been computed for the composite laminates defined in Table 1.

Hereinafter, glass/epoxy and carbon/epoxy refer to the materials defined in this table. This plot demonstrates that both Young's moduli are very close as expected.

Defining the maximum admissible longitudinal strain in a unidirectional (UD) lamina on the plane perpendicular to the fibre direction as

$$Y_{et} = \frac{Y_t}{E_{22}}, \quad (4)$$

the final expression of the stress criterion (2) takes the form:

$$\frac{\varepsilon_{yy}}{Y_{et}} \geq \frac{E_{22}}{E_{22}}, \quad (5)$$

where $\frac{E_{22}}{E_{22}} \geq 1$ according to Fig. 2. This inequality represents a necessary condition for rupture at any point in the inner 90° lamina. It can be noticed from the above expression that the stress criterion will be fulfilled for all the points of the 90° lamina for the same value of the applied strain ε_{yy} because the right-hand side does not depend on the point (x, y, z) . Under present assumptions and according to this conclusion, stress criterion does not impose any restriction on the size of the originated crack because the stress criterion is either simultaneously fulfilled either at all points or at none point of the 90° lamina.

3. Energy criterion

The energy criterion adopted here is based on the incremental Griffith criterion proposed by several authors as Garrett and Bailey (1977); Hashin (1996); Leguillon (2002) and Cornetti et al. (2006, 2012). The energy balance of an abrupt onset of a crack under the assumptions presented above can be expressed as

$$\Delta\Pi + \Delta E_k + G_c 2\Delta a = 0, \quad (6)$$

where $\Delta\Pi$ and ΔE_k are, respectively, the change in elastic potential and kinetic energy due to crack onset. $G_c 2\Delta a$ is the energy dissipated during the onset. G_c and Δa are, respectively, the fracture toughness, also referred to as fracture energy, and the semilength of the crack after the onset. In the present problem, a constant fracture toughness G_c during the crack onset and growth can be assumed, because the transverse crack in the 90° ply grows in pure mode I, therefore $G_c = G_{Ic}$. Although the pure fracture mode I is assumed for a crack growing in a plane without breaking fibres, different values for the fracture toughness might be considered for the crack growth along the direction parallel or perpendicular to the fibres. Nevertheless, in the present work both values are assumed to be very close and are identified with the transverse fracture toughness for transverse-crack propagation-direction perpendicular to the fibres. Additionally it is assumed that no damage at the interface takes place. The crack growth after reaching the 90° and 0° ply interface is not studied here.

As the initial state is quasistatic, then there is a production of kinetic energy $\Delta E_k \geq 0$ and the above expression leads to

$$-\Delta\Pi \geq G_c 2\Delta a. \quad (7)$$

Introducing in this inequality the relation between the value of the energy release rate (ERR) G and the derivative of the potential energy with respect to the crack length $2a$, known from the Linear Elastic Fracture Mechanics (LEFM) theory, $G(a) = -d\Pi/d(2a)$, implying that

$$-\Delta\Pi = 2 \int_0^{\Delta a} G(a) da \quad (8)$$

the energy balance in (6) finally leads to

$$\int_0^{\Delta a} G(a) da \geq G_c \Delta a. \quad (9)$$

This expression is a relation between the elastic energy available to be released during the crack onset (left-hand side) and the energy necessary for the abrupt onset (right-hand side). Values for G are required to fulfill the criterion (9). Unfortunately any analytical solution of the problem with cracked geometry, see Fig. 1(b), is not available. Thus, G is obtained by computational methods.

Values of G have been calculated by Blázquez et al. (2009) for a particular configuration of the problem by means of the Boundary Element Method (BEM). These results can be exploited for many configurations of the present problem by means of dimensional analysis. Following García (2014), a dimensionless ERR \hat{G} can be defined by the relation:

$$G = E_{22} \varepsilon_{yy}^2 t_{90} \hat{G}\left(\hat{a}, \frac{t_0}{t_{90}}, \text{E.P.}\right), \quad (10)$$

where $\hat{a} = a/t_{90}$ and “E.P.” are the dimensionless elastic properties of the lamina. Thanks to this expression, the number of models to be solved is reduced drastically since the values obtained can be exploited for any values of E_{22} , ε_{yy} and t_{90} .

Values of G obtained by Blázquez et al. (2009) are normalised here according to (10) and plotted in Fig. 3. This figure represents the values of $\hat{G}(\hat{a})$ for $t_0/t_{90} = 1$ and glass/epoxy versus \hat{a} .

In view of Fig. 3, three regions with different behaviour of $\hat{G}(\hat{a})$ can be differentiated:

1. In the first region (I) where the crack is short enough, $\hat{G}(\hat{a})$ is approximately linear. The reason is that when the crack tip is far enough from the interface, \hat{G} can be approximated by its value corresponding to an infinite cracked plate under a remote uniaxial loading. Thus, at this extreme, \hat{G} has an asymptotic linear behaviour
2. The second region (II) can be considered around the crack length $\hat{a}_{\max, \hat{G}}$ where the function $\hat{G}(\hat{a})$ reaches its maximum value. In this region, the influence of the stiffer material in the 0° ply cannot be neglected.
3. In the third region (III), there is a strong transition between the maximum and the necessary zero value of $\hat{G}(\hat{a})$ for $\hat{a} = 1$ governed by the asymptotic singular behaviour of $\hat{G}(\hat{a} \rightarrow 1)$, conditioned by the dominating influence of the outer ply.

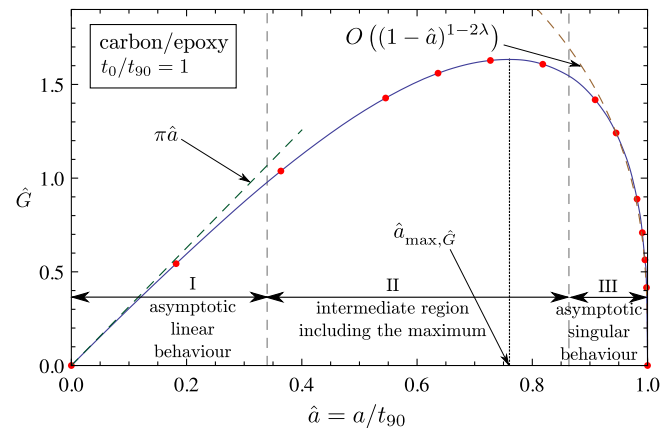


Fig. 3. Dimensionless ERR \hat{G} of the transverse crack in the inner ply for carbon/epoxy as a function of its dimensionless length $\hat{a} = a/t_{90}$.

At this extreme, where the length of the transverse crack is close to the thickness of the inner-ply ($\hat{a} \rightarrow 1$), jump of the mechanical properties (becoming stiffer) across the interface implies that $\hat{G} \rightarrow 0$ for $\hat{a} \rightarrow 1^-$. In the present particular problem configuration, the asymptotic behaviour was described by Blázquez et al. (2008) as

$$\hat{G} = O((1 - \hat{a})^{1-2\lambda}) \quad \text{for } \hat{a} \rightarrow 1^- \quad (12)$$

where λ is the stress singularity order of the trimaterial corner defined by the transverse crack terminating at the interface. The value of λ for carbon/epoxy $\lambda = 0.330111$ has been computed by the analytic procedure developed by Barroso et al. (2003).

The position of the maximum of the curve is affected by the ratio E_{22}/E_{11} . The maximum moves towards the interface, when the ratio E_{22}/E_{11} increases, diminishing the region III affected by the jump in stiffness across the interface.

Fig. 4 shows the variations of \hat{G} for glass/epoxy computed by the same BEM code. Unlike the carbon/epoxy laminate with $E_{22}/E_{11} = 0.0678$, the glass/epoxy laminate has a ratio $E_{22}/E_{11} = 0.33$. This difference in values of E_{22}/E_{11} generates the effect predicted above: the crack length corresponding to the maximum of ERR is situated closer to the outer-ply because the influence of this ply is lower.

The influence of the ratio of the inner and outer ply thickness, t_0/t_{90} , can also be observed in Fig. 4. Values of the parameter t_0/t_{90} shown in this figure correspond with the laminate configurations tested by Parvizi et al. (1978). On the one hand, this figure shows that the maximum value of \hat{G} grows and slightly moves toward the interface when $t_0/t_{90} \rightarrow 0$. In the limit case, where the outer ply does not exist ($t_0/t_{90} = 0$), the value of \hat{G} increases to infinity for $\hat{a}/t_{90} \rightarrow 1$. On the other hand, Fig. 4 shows that for $t_0/t_{90} \rightarrow \infty$, which corresponds to very thick outer plies, the values of \hat{G} tend to a limit curve. The reason for this is that for large values of t_0/t_{90} , the influence of an addition of material to the outer ply (corresponding to an increase of the value of t_0/t_{90}) on the behaviour of the crack in the inner ply is negligible because the tip of the crack is far from the new material added (due to the large value of t_0/t_{90}).

Introducing the expression of \hat{G} (10) in the expression of the incremental energy criterion (9) gives, after a rearrangement,

$$\frac{\varepsilon_{yy}^2 E_{22} t_{90}}{G_c} \geq g(\Delta \hat{a}), \quad (13)$$

where g is a dimensionless positive function defined as

$$g\left(\Delta \hat{a}; \frac{t_0}{t_{90}}, \text{E.P.}\right) = \frac{\Delta \hat{a}}{\int_0^{\Delta \hat{a}} \hat{G}(\hat{a}) d\hat{a}}. \quad (14)$$

Some arguments of g will be omitted in the following for the sake of simplicity. The function g can be understood as the dimensionless ratio of the necessary to available energy for the onset of a crack of semilength $\Delta \hat{a}$. It means, function g measures the “resistance against crack onset”, from an energetic point of view, for the crack semilength $\Delta \hat{a}$ at the onset. Fig. 5(a) shows the function $g(\Delta \hat{a})$ for carbon/epoxy.

Apparently, looking at Fig. 5(a) by a naked eye, the function $g(\Delta \hat{a})$ is decreasing in the whole interval $(0, 1)$. However, as will be seen, g has a quite shallow minimum at $\Delta \hat{a}_{\min, g} < 1$ typically close to 1. Thus, g is decreasing for a major part of the interval $(0, 1)$, and generally speaking a lower applied strain is necessary for larger lengths of the crack at the onset. Fig. 5(b) shows a detailed plot of g around its minimum very close to the interface between the plies. This result could be erroneously attributed to some errors in BEM results or in their interpolation. However, this property of the universal function g can be demonstrated independently of the computational results. Moreover, this minimum is situated closer to the interface than the maximum of \hat{G} . Both properties are demonstrated by García (2014).

After a rearrangement of (13), the condition for the onset given by the energy criterion can be rewritten in an analogous manner to that defined by the stress criterion (5) as

$$\varepsilon_{yy} \geq \sqrt{\frac{G_c}{E_{22} t_{90}}} g(\Delta \hat{a}). \quad (15)$$

Note that, unlike the stress criterion, the energy criterion defines a condition for the strain required for the onset which depends on the semilength of the crack at the onset $\Delta \hat{a}$.

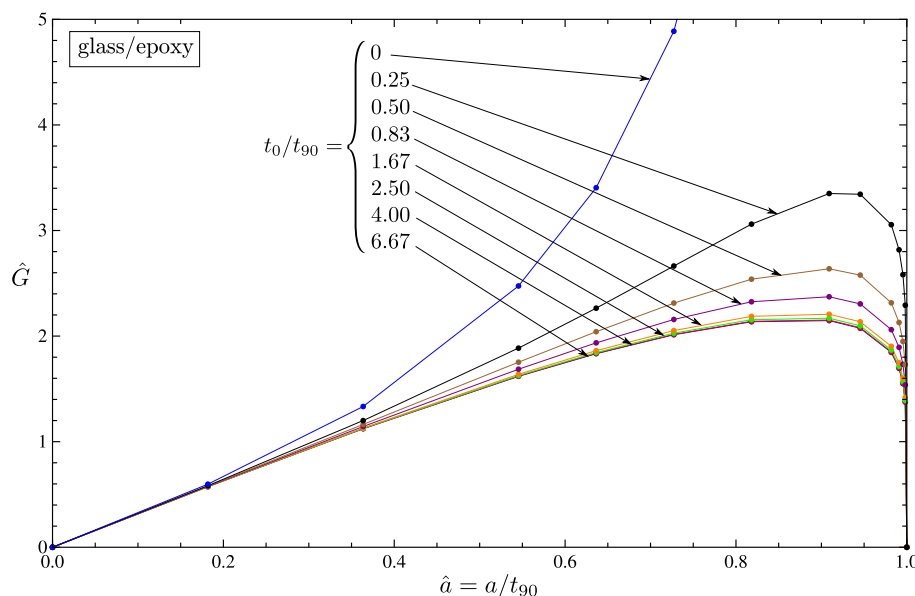


Fig. 4. Dimensionless ERR \hat{G} of the transverse crack in the inner ply as a function of its dimensionless length $\hat{a} = a/t_{90}$ for glass/epoxy and laminates used by Parvizi et al., 1978.

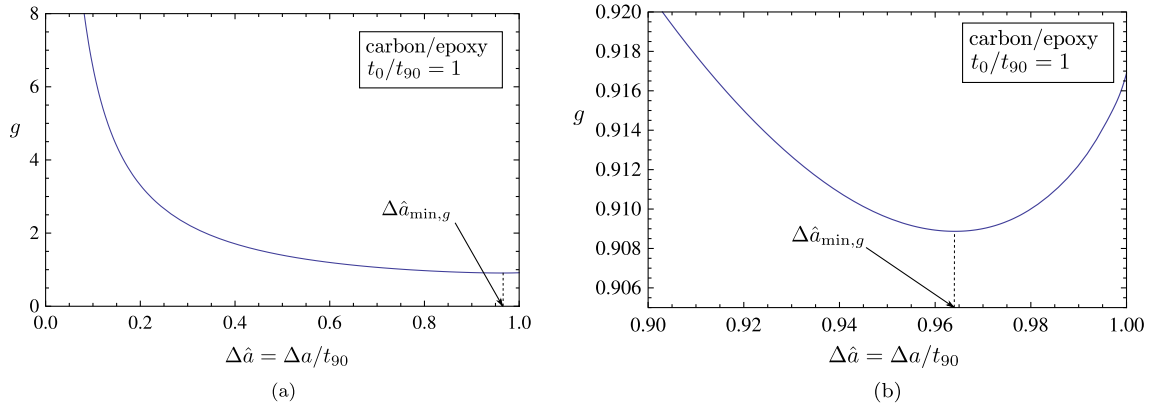


Fig. 5. Example of the universal dimensionless function $g(\Delta\hat{a})$ for carbon/epoxy and $t_0/t_{90} = 1$ plotted (a) in its whole domain and (b) in the vicinity of $\Delta\hat{a}_{\min,g}$.

4. Coupled stress and energy criterion

In this section the coupled criterion by Leguillon (2002) will be applied to the present problem following the procedure used by Mantić (2009) and Mantić and García (2012). The key idea of the coupled criterion, referred to as Leguillon's hypothesis, assumes that a sufficient condition for the transverse crack onset is the simultaneous fulfilment of the two necessary conditions described in previous sections.

Both stress criterion in (5) and energy criterion in (15) define a required value for the applied strain ε_{yy} which can originate a crack. In order to have both conditions expressed for the same magnitude, (15) is divided by the critical strain transverse to the fibres Y_{et} , see (4). Thus, an expression of the energy criterion comparable to the expression of the stress criterion is obtained

$$\frac{\varepsilon_{yy}}{Y_{et}} \geq \frac{1}{Y_{et}} \sqrt{\frac{G_c}{E_{22}t_{90}}} g\left(\Delta\hat{a}, \frac{t_0}{t_{90}}, \text{E.P.}\right), \quad (16)$$

Following Mantić (2009), a dimensionless brittleness number γ can be introduced as

$$\gamma = \frac{1}{Y_t} \sqrt{\frac{E_{22}G_c}{t_{90}}}. \quad (17)$$

Note that γ is a structural parameter dependent on some material properties and a geometry parameter. Then, the final expression of the energetic criterion adopts the form

$$\frac{\varepsilon_{yy}}{Y_{et}} \geq \gamma \sqrt{g\left(\Delta\hat{a}, \frac{t_0}{t_{90}}, \text{E.P.}\right)}. \quad (18)$$

Assuming Leguillon's hypothesis, the critical strain leading to the crack onset is given by the minimum value of ε_{yy} which fulfills both criteria. Fig. 6 shows the curves of both criteria, (5) and (18), for several values of the dimensionless structural parameter γ , for carbon/epoxy and $t_0/t_{90} = 1$. This figure shows that there are two possible scenarios depending on the value of γ :

- Scenario A: This scenario corresponds to the values of γ for which the stress and energy criteria curves have some common point. In this case, the critical strain ε_{yy}^c and the length of the crack $\Delta\hat{a}_c$ are determined by this common point. In fact, as the stress criterion is a horizontal straight line, ε_{yy}^c does not depend on $\Delta\hat{a}_c$. Consequently, in scenario A, ε_{yy}^c will be defined by the constant value imposed by the stress criterion,

$$\frac{\varepsilon_{yy}^{c,A}}{Y_{et}} = \frac{E_{22}}{\bar{E}_{22}}. \quad (19)$$

Thus, for the values of γ leading to scenario A, the critical strain depends only on the values of E_{22}/\bar{E}_{22} and Y_{et} . However, the extension of the crack length at the onset does depend on the value of γ and can be determined by finding the common point of the two criteria curves

$$\gamma \sqrt{g(\Delta\hat{a}_c^A)} = \frac{E_{22}}{\bar{E}_{22}}, \quad (20)$$

where $\Delta\hat{a}_c^A$ is obtained by the resolution of this non-linear equation.

- Scenario B: This scenario corresponds to the values of γ for which no common point exists between the stress and energy criteria curves. In this case, as can be seen in Fig. 6, the stress criterion is fulfilled for a value of the applied strain lower than the minimum value defined by the energy criterion corresponding to its value at $\Delta\hat{a} = \Delta\hat{a}_{\min,g}$. Consequently, the coupled criterion will be fulfilled when the energy criterion is fulfilled. Therefore, scenario B is, in fact, governed only by the energy criterion. The lower value of the critical strain imposed by the energy criterion is determined by the minimum value calculated above

$$\frac{\varepsilon_{yy}^{c,B}}{Y_{et}} = \gamma \sqrt{g(\Delta\hat{a}_{\min,g})}, \quad (21)$$

where the value $\Delta\hat{a}_{\min,g}$ has been defined in Section 3. Since g does not depend on γ , therefore neither $\Delta\hat{a}_{\min,g}$ does. As a consequence $g(\Delta\hat{a}_{\min,g})$ is a constant which depends only on the dimensionless elastic properties and on the relation between layer thicknesses t_0/t_{90} . For given values of these parameters, the value of $g(\Delta\hat{a}_{\min,g})$ becomes a constant and consequently a linear relation exists between the critical strain $\varepsilon_{yy}^{c,B}$ and the value of γ , see (21). This is an important consequence because it reduces the calculus of this dependence to the minimisation of the function g to obtain $\Delta\hat{a}_{\min,g}$. Having computed $g(\Delta\hat{a}_{\min,g})$, the value of the critical strain for any value of γ can be obtained by (21).

The value of the critical length of the debond $\Delta\hat{a}_c$ is determined directly by the minimum value for the energetic criterion. Therefore

$$\Delta\hat{a}_c^B = \Delta\hat{a}_{\min,g}, \quad (22)$$

a value which is independent of the structural parameter γ .

Both scenarios are separated by a threshold value of γ , denoted as γ_{th} , for which both criteria curves are tangent. Hence,

$$\gamma_{th} = \frac{\frac{E_{22}}{\bar{E}_{22}}}{\sqrt{g(\Delta\hat{a}_{\min,g})}}. \quad (23)$$

Then, according to above definitions and as can also be seen in Fig. 6, scenario A corresponds to $0 \leq \gamma \leq \gamma_{th}$ and scenario B

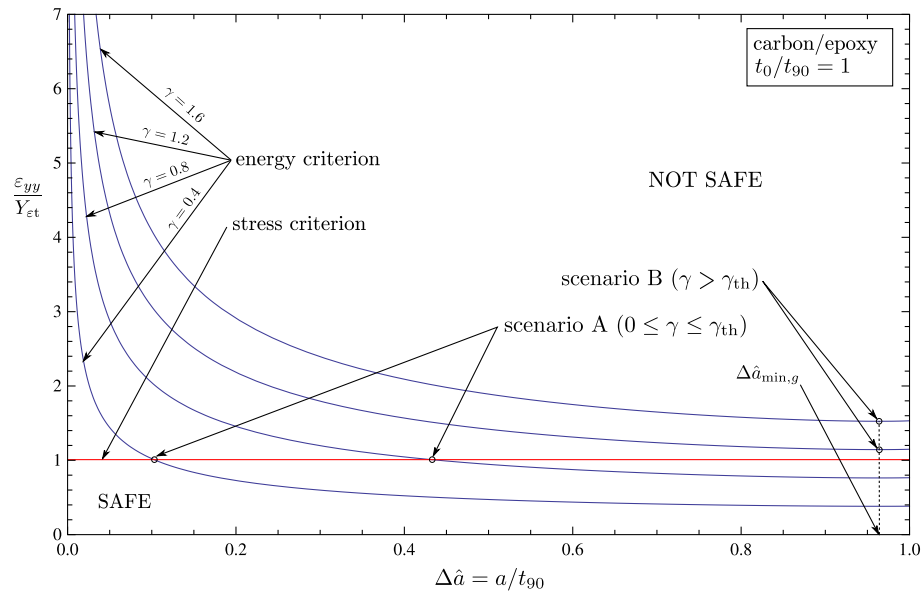


Fig. 6. Coupled stress and energy criteria for several values of the structural parameter γ for carbon/epoxy laminates with $t_0/t_{90} = 1$.

corresponds to $\gamma > \gamma_{th}$. Therefore, in view of (17), scenario A, where the critical strain corresponds approximately to $Y_{\epsilon t}$, is associated to low values of G_c and E_{22} , to high values of Y_t and to thick laminates.

Combining both scenarios, the critical strain leading to the first transverse crack onset is given by,

$$\frac{\epsilon_{yy}^c}{Y_{\epsilon t}} = \max \left(\frac{E_{22}}{\bar{E}_{22}}, \gamma \sqrt{g(\Delta \hat{a}_{min,g})} \right). \quad (24)$$

Variation of ϵ_{yy}^c as a function of the dimensionless structural parameter γ , for carbon/epoxy and $t_0/t_{90} = 1$, is shown in Fig. 7. This plot as well as expression in (24) show the extreme simplicity of the critical strain prediction if the geometrical parameter t_0/t_{90} is fixed. From the computational model, only a scalar value $\sqrt{g(\Delta \hat{a}_{min,g})}$, independent of the strength and fracture toughness properties, is necessary to obtain this curve.

These results predict that the transverse-ply can support strains quite higher than its nominal ultimate strain $Y_{\epsilon t}$. The reason is that the stress criterion is not a sufficient condition but just a necessary one for the crack onset.

The progressive damage models of laminates (see e.g. Matzenmiller et al., 1995) predict a failure of the 90° ply using a stress or equivalent strain criterion. However, according to the present results, under the hypothesis of the first failure associated to a transverse crack in the 90° ply, the first failure of the laminate is not expected to appear for nominal ultimate strain value in the direction perpendicular to the fibres, $Y_{\epsilon t}$, for values of $\gamma > \gamma_{th}$. In these cases, predictions based on the traditional stress (or strain) criteria can become too conservative because the critical strain can become significantly larger than $Y_{\epsilon t}$, as can be seen in Fig. 7.

The critical length of the crack at the onset $\Delta \hat{a}_c$ is shown in Fig. 8 as a function of γ for carbon/epoxy. This figure shows that $\Delta \hat{a}_c$ is

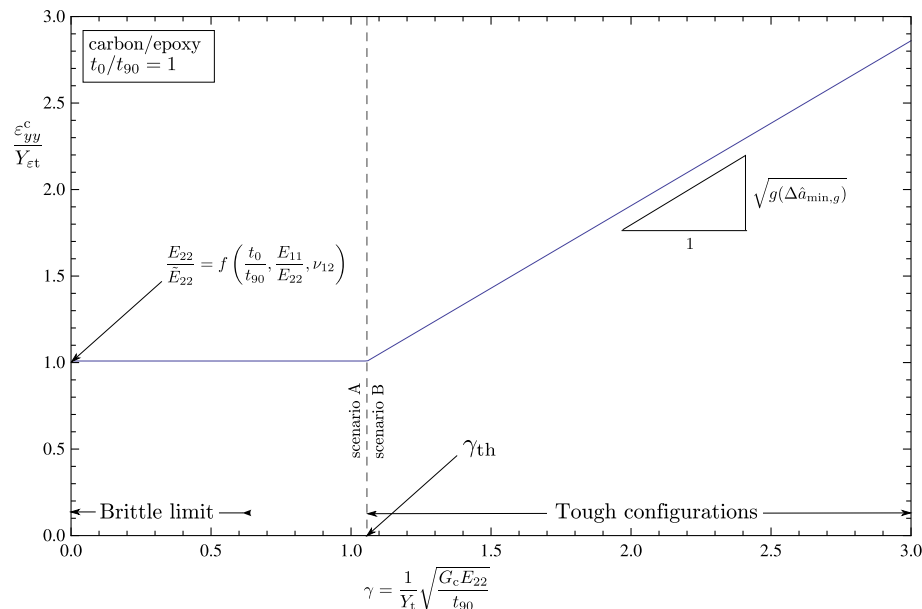


Fig. 7. Critical applied strain ϵ_{yy}^c , normalised by the ultimate transverse strain $Y_{\epsilon t}$ of the lamina, as a function of the structural parameter γ , for carbon/epoxy and $t_0/t_{90} = 1$.

larger for higher values of γ and that it is constant in scenario B. In this scenario, $\Delta\hat{a}_c$ corresponds to about 96.4% of the inner-ply thickness for carbon/epoxy. This value corresponds, as has been described above, to the position of the minimum value of the function $g, \Delta\hat{a}_{\min,g}$. This position depends strongly on the relation between the longitudinal and transverse Young's moduli: E_{22}/E_{11} and the ply-thickness ratio t_0/t_{90} . A lower value of E_{22}/E_{11} corresponds to a larger distance between the crack tip at the crack onset and the interface.

The reason for this is that a stiffer outer ply makes more difficult approaching the interface by a crack. For instance, in the case of the glass/epoxy lamina (that corresponds normally to a lower difference between transverse and longitudinal properties than a carbon/epoxy lamina), the minimum of the function g corresponds to a 99.83% of the inner-ply thickness for $t_0/t_{90} = 0.83$.

5. Post crack-onset evolution

The length of the crack at the onset, $\Delta\hat{a}_c$, predicted by the coupled criterion is very difficult to verify experimentally. The reason is that this original crack could grow unstably after the onset, for a fixed $\varepsilon_{yy} = \varepsilon_{yy}^c$. Therefore, a visible crack in a later post-failure exploration of the specimen, could be larger than predicted by $\Delta\hat{a}_c$, its final length being referred to as the dimensionless arrest length of the crack, \hat{a}_a .

Following the proof proposed by Mantič (2009), in this case it is possible to demonstrate that $G(\Delta\hat{a}_c) > G_c$ for $\gamma < \gamma_{th}$. Therefore, the originated crack will grow unstably after the onset for $\gamma < \gamma_{th}$ to the so-called arrest length where $G(\hat{a}_a) = G_c$ and $G'(\hat{a}_a) < 0$. The proof begins by evaluating the logarithmic derivative of the function $g(\Delta\hat{a})$ as

$$\frac{d \log g}{d \Delta\hat{a}} = \frac{1}{g(\Delta\hat{a})} \frac{dg}{d \Delta\hat{a}} = \frac{G_c}{G_c \Delta\hat{a}} - \frac{G(\Delta\hat{a})}{\int_0^{\Delta\hat{a}} G d\hat{a}}. \quad (25)$$

As discussed previously, in both scenarios energy criterion is fulfilled as an equality, then it holds

$$G_c \Delta\hat{a}_c = \int_0^{\Delta\hat{a}_c} G d\hat{a}. \quad (26)$$

From the monotony analysis of $g(\Delta\hat{a})$,

$$\frac{dg}{d \Delta\hat{a}} < 0 \text{ for } \Delta\hat{a} < \Delta\hat{a}_{\min,g} \text{ and } \frac{dg}{d \Delta\hat{a}} = 0 \text{ for } \Delta\hat{a} = \Delta\hat{a}_{\min,g}. \quad (27)$$

Introducing conditions (26) and (27) and $g > 0$ in the expression of the logarithmic derivative (25), the next outcome is obtained

$$G(\Delta\hat{a}_c) > G_c \text{ for } \Delta\hat{a}_c < \Delta\hat{a}_{\min,g}, \text{ and } G(\Delta\hat{a}_c) = G_c \text{ for } \Delta\hat{a}_c = \Delta\hat{a}_{\min,g}. \quad (28)$$

The first result implies that the crack grows unstably after the onset for $\gamma < \gamma_{th}$. The second result implies that it is necessary to evaluate the derivative of G with respect to the crack length $2a$ to know the post-onset evolution for $\gamma \geq \gamma_{th}$.

Taking into account that $g'(\Delta\hat{a}_{\min,g}) > 0$, and because g has a minimum value at $\Delta\hat{a}_{\min,g}$, it can be demonstrated that $G'(\Delta\hat{a}_{\min,g}) < 0$, see a proof by García (2014). Combining this expression with (28), it is obtained that no unstable growth is expected for $\gamma \geq \gamma_{th}$. Hence, for these values of γ ,

$$\hat{a}_a^B = \Delta\hat{a}_c^B = \Delta\hat{a}_{\min,g}. \quad (29)$$

It is useful to observe that for $\gamma < \gamma_{th}$, after the initial crack onset, the value of the arrest length \hat{a}_a of the crack can be calculated as

$$G(\hat{a}_a, \varepsilon_{yy}^c) = G_c \quad \text{and} \quad G'(\hat{a}_a, \varepsilon_{yy}^c) < 0. \quad (30)$$

where ε_{yy}^c is the critical strain generating the crack onset. The value of the arrest length of the crack for both scenarios has been computed and the results are also shown in Fig. 8 versus γ for carbon/epoxy. This plot shows that the developed theoretical model predicts that the arrest length is always very close to the interface between the inner-ply and the outer-ply. This observation is demonstrated in general by García (2014). In particular, for carbon/epoxy and any value of γ

$$\hat{a}_a \geq \Delta\hat{a}_{\min,g} \left(\frac{t_0}{t_{90}} = 1, \text{E.P.} \right) \approx 96.4\%, \quad \forall \gamma. \quad (31)$$

In view of this result, the distance between the crack tip and the interface may be only a few times larger than the fibre radius, cf. París et al. (2010b). Thus, the hypothesis of homogeneous material for the inner-ply might not be *a priori* coherent because of the influence of its inhomogeneity on the crack growth. Outcomes of

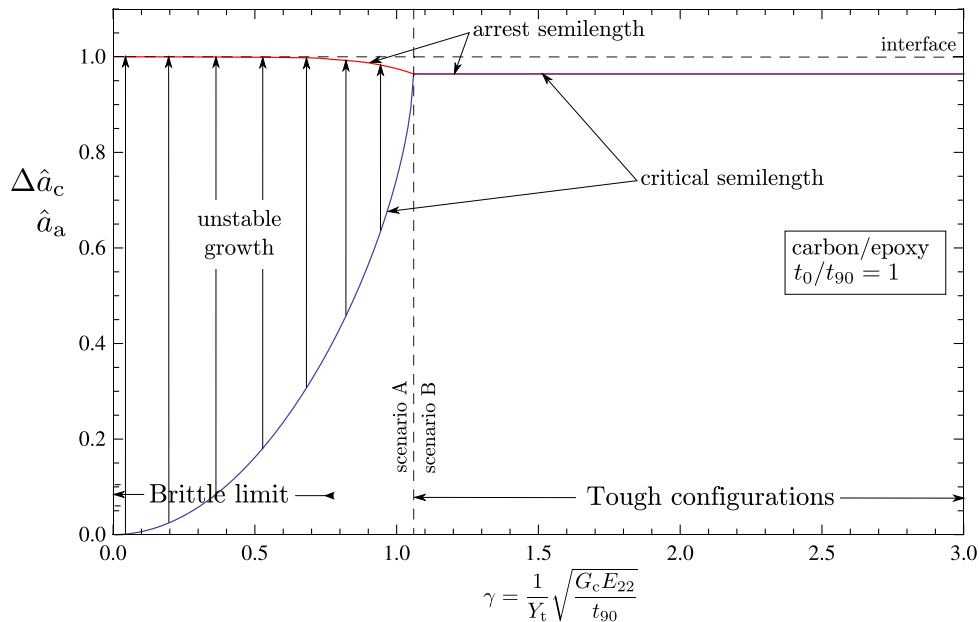


Fig. 8. Critical length $\Delta\hat{a}_c$ and arrest length \hat{a}_a as a function of the structural parameter γ for carbon/epoxy and $t_0/t_{90} = 1$.

this observation will not be studied here but it is important to take it into account when discussing the range of validity of the results presented.

6. Effect of the laminate geometry

Results presented above depend on the two geometry parameters of the laminate: t_{90} and t_0/t_{90} . It is interesting to study their effect on the results predicted. In particular, the variation of the critical strain originating a crack as a function of the geometry can have some interesting consequences for the design of laminates.

6.1. Influence of the inner-ply thickness for a fixed t_0/t_{90} –size effect

A size effect in the problem is associated to the dependency of results on γ as shown in Fig. 7. According to the definition of γ in (17), the inner-ply thickness is the only material-independent parameter in γ . Following this idea, a reference length $t_{90,r}$ can be defined in terms of the material properties which appear in the definition of γ , i.e.

$$t_{90,r} = \frac{G_c E_{22}}{Y_t^2}. \quad (32)$$

Thus, γ can be rewritten as a function of t_{90} and $t_{90,r}$

$$\gamma = \sqrt{\frac{t_{90,r}}{t_{90}}}. \quad (33)$$

Hence, a threshold thickness given by the threshold value of γ can be defined as

$$t_{90,th} = \frac{t_{90,r}}{\gamma_{th}^2}. \quad (34)$$

This threshold thickness $t_{90,th}$ has a very important physical meaning. This value separates two ranges of the inner-ply thickness with a very different behaviour. Keeping fixed $t_{90,r}$ (which corresponds to keep fixed a relation of E_{22} , Y_t and G_c), a critical strain ε_{yy}^c close to the ultimate UD transverse strain Y_{et} of the lamina is predicted for laminates with $t_{90} > t_{90,th}$ (corresponding to $\gamma < \gamma_{th}$). However, a value of strain significantly higher than the nominal value Y_{et} of the lamina is expected to be withstand by laminates with a lower inner-ply thickness than the threshold value, $t_{90} < t_{90,th}$. Therefore, a size effect associated to the inner-ply thickness is predicted by the present theoretical model. In fact, this size effect is strong, as can be seen when the results are presented as a function of γ in Fig. 7 and as a function of the dimensionless inner-ply thickness $t_{90}/t_{90,r}$ in Fig. 9. Representation of these results as a function of $t_{90}/t_{90,r}$ shows that the change of the apparent strength of the inner-ply is very abrupt. Fig. 9 shows the importance, discussed above, of the threshold value of the inner-ply thickness $t_{90,th}$. Qualitatively similar curves were presented by Leguillon (2002); Ladevèze et al. (2006).

Taking into account the expression (33), it is possible to express the critical strain in scenario B as a function of the dimensionless inner-ply thickness

$$\frac{\varepsilon_{yy}^c}{Y_{et}} = \sqrt{\frac{t_{90,r}}{t_{90}}} \sqrt{g(\Delta \hat{a}_{min,g})}. \quad (35)$$

Therefore, in view of this result, in scenario B, the critical strain is strongly increasing for $t_{90}/t_{90,r} \rightarrow 0^+$.

Physical interpretation of this abrupt change in the slope and curvature of the critical strain curve in Fig. 9 is due to two different reasons. The first one is the change of criterion governing the failure as described above. The second one is the size effect inherent to

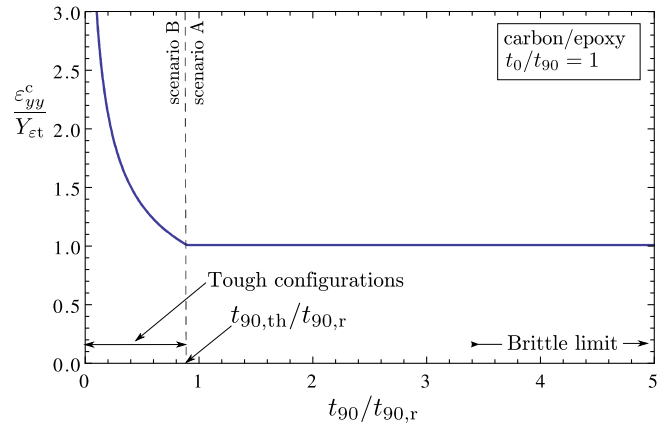


Fig. 9. Critical strain ε_{yy}^c normalised by the unidirectional ultimate UD transverse strain Y_{et} as a function of the inner-ply thickness t_{90} normalised by the reference thickness $t_{90,r}$ for carbon/epoxy and $t_0/t_{90} = 1$.

the energy criterion. Introducing the dimensionless expression of $G(a)$ from (10) in the inequality of the energy criterion in (9) gives,

$$E_{22} \varepsilon_{yy}^2 t_{90}^2 W \int_0^{\Delta \hat{a}} \hat{G}(\hat{a}) d\hat{a} \geq G_c t_{90} W \Delta \hat{a}. \quad (36)$$

The left-hand side represents the released energy when an onset of half-length Δa appears. In scenario B, where exclusively the energy criterion governs the crack onset, half-length of the crack at the onset is fixed $\Delta a_c = \Delta \hat{a}_{min,g} t_{90} \sim t_{90}$. As a consequence, the integral term in (36) is fixed and the change in potential energy

$$-\Delta \Pi \sim E_{22} \varepsilon_{yy}^2 t_{90}^2 W, \quad (37)$$

where W is the width of the laminate. In view of this result, released energy is proportional to the product of stress $E_{22} \varepsilon_{yy}$ and strain ε_{yy} and to the volume $t_{90}^2 W$ of the material where the energy is released. In the case of the energy dissipated due to the onset,

$$\Delta \Gamma \sim G_c t_{90} W, \quad (38)$$

thus, it is proportional to the fracture toughness and the new crack surface created after the onset. In view of the above results and unlike the released energy, which is released in a volume, the energy necessary for the crack onset is dissipated at a surface. This is actually the origin of the size effect. When the laminate is thinner, then the volume where the available energy is stored near the crack ($\sim t_{90}^2 W$) decreases faster than the surface where the energy will be dissipated ($\sim t_{90} W$). Therefore, an increase of strain ε_{yy}^c is necessary to compensate this effect.

6.2. Influence of the geometric parameter t_0/t_{90}

The influence of t_0/t_{90} is pointed out by some evidences:

- Expression of the stress criterion depends directly on the value of t_0/t_{90} by the expression of E_{22}/\bar{E}_{22} (3). In the following, this dependence is studied by evaluating the value of E_{22}/\bar{E}_{22} at the extremal values of t_0/t_{90} .

At the extreme $t_0/t_{90} \rightarrow 0^+$ which corresponds to a laminate without the longitudinal ply, it is fulfilled that

$$\lim_{t_0/t_{90} \rightarrow 0^+} \frac{E_{22}}{\bar{E}_{22}} = 1. \quad (39)$$

Therefore, the stress criterion (5) for the extreme case without outer ply ($t_0/t_{90} = 0$) leads to a foreseeable result:

$$\frac{\varepsilon_{yy}}{Y_{et}} \geq 1, \quad (40)$$

which corresponds to the typical prediction for an unidirectional transverse lamina.

At the other extreme, when $t_0/t_{90} \rightarrow \infty$, then the outer-ply is much thicker than the inner-ply, the next result is obtained for the stress criterion, assuming $E_{22} < E_{11}$ and $\nu_{12}^2 \leq 1$

$$\lim_{t_0/t_{90} \rightarrow \infty} \frac{E_{22}}{\bar{E}_{22}} = \frac{1 - \frac{E_{22}}{E_{11}} \nu_{12}^2}{1 - \nu_{12}^2} > 1. \quad (41)$$

Hence, the effect of a large thickness of the outer-ply compared to the inner-ply thickness is to increase the apparent strength in the stress criterion of the inner-ply. This increase depends strongly on the lamina properties ν_{12} and E_{22}/E_{11} . For example, the limit in the expression (41) is equal to 1.0558 for glass/epoxy and 1.092 for carbon/epoxy. In general, for typical values of ν_{12} , this limit value will be quite close to the unity. Therefore, we can conclude from these results that the influence of the outer ply in the stress criterion is quite limited for the majority of usual composites.

- Energy criterion depends on the value of t_0/t_{90} through the dependence of \bar{G} on it, see (10) and Fig. 4. In general, a pair of thinner outer plies makes the energy to be released in an easy way when the crack grows. Thus, the universal dimensionless function g is expected to depend on this parameter as well. However, Fig. 10 shows that the variation of g with t_0/t_{90} is not significant for the values of the geometric parameter t_0/t_{90} tested by Parvizi et al. (1978). Consequently, critical strain predicted does not change significantly either. Fig. 11 confirms this result for the range of t_0/t_{90} tested by Parvizi et al. (1978).

As can be expected from the influences described above for the two independent criteria, t_0/t_{90} introduces a small variation of the critical strain predicted by the coupled criterion. Figs. 11 and 12 show the critical strain originating a crack as a function of, respectively, $2t_{90}$ and γ for the values of t_0/t_{90} tested by Parvizi et al. (1978). In both scenarios, a higher t_0/t_{90} increases the critical strain originating a crack. The reason is that in scenario A, ε_{yy}^c is determined directly by the stress criterion which is slightly more restrictive for higher values of t_0/t_{90} as demonstrated above. On the other hand, in scenario B, ε_{yy}^c is determined by the energy criterion which is also a bit more restrictive for higher values of t_0/t_{90} because of the reasons presented above.

In summary, the effect of an increase of t_0/t_{90} is a slight increase of the critical strain that originates a crack in the inner-ply.

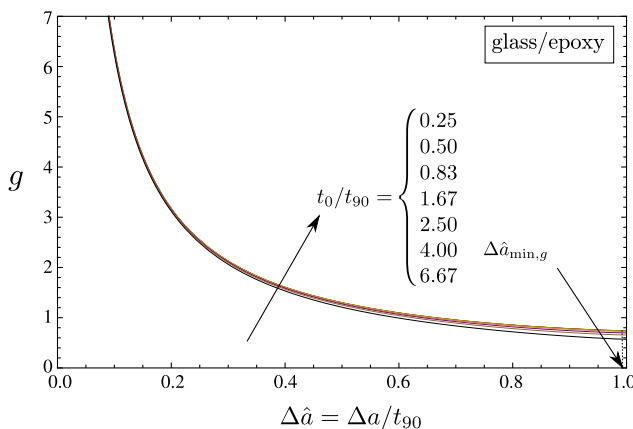


Fig. 10. Universal dimensionless function $g(\Delta\hat{a})$ for glass/epoxy and the laminates used by Parvizi et al., 1978. The arrow shows the direction of increasing values of t_0/t_{90} in the list.

7. Comparison with experimental evidences

Two important and experimentally verifiable results have been obtained from the present theoretical model:

- A strong size effect of the inner-ply thickness on the critical strain ε_{yy}^c leading to the first crack onset.
- Arrest length of the crack originated in the inner-ply is very close to the inner-ply thickness.

Whereas the first prediction will be compared with experimental results obtained by Parvizi et al. (1978), the second result will be verified qualitatively by the observation of the cracks found in the specimens tested by París et al. (2010b).

7.1. Experimental evidences for the size effect in the critical strain

Results for the critical strain that originates the initial failure in a cross-ply laminate are compared with the experimental results by Parvizi et al. (1978). They tested glass/epoxy cross-ply laminates (epoxy resin Shell Epikote 828 reinforced with E-glass fibre rovings Silenka 1200 tex). These laminates were made with a transverse-ply thickness ranging from 0.1 to 4 mm, keeping constant the longitudinal-ply thickness at 0.5 mm. Note that these laminates are not geometrically similar laminates since t_0/t_{90} is not fixed. Therefore, the theoretical model developed here does not predict exactly a two-straight-lines law for ε_{yy}^c versus γ as shown in Fig. 12.

To apply the model, elastic properties are taken from the information given by Parvizi et al. (1978). However, the authors did not give all the elastic properties necessary to characterise the laminate in accordance with our model. In particular, no data about the Poisson's coefficients are available in their article. As a consequence, the Poisson's coefficients are taken from Nurhaniza et al. (2010) who use a very similar material. A collection of the elastic properties has been presented in Table 1.

On the other hand, both the transverse fracture toughness G_c and the unidirectional ultimate UD transverse strain Y_{at} are experimentally obtained by Parvizi et al. (1978) as, $G_c = 240 \pm 60 \text{ J/m}^2$, and $Y_{at} = 0.005 \pm 0.001$, which corresponds to $Y_t = 70 \pm 14 \text{ MPa}$.

First, a comparison based on the central values of the parameters G_c and Y_{at} is carried out. Subsequently, a comparison taking the whole range of validity into account is done to observe the variability of results. Both comparisons have been calculated neglecting the residual stresses due to the different temperature of curing and service. Generally, in glass/epoxy laminates, residual stresses are much lower than in carbon/epoxy laminates due to the following reasons: lower contrast between the longitudinal and transverse properties and the same sign of the dilatation coefficient for the longitudinal and transverse direction in glass/epoxy laminates. In addition, the laminates used by Parvizi et al. (1978), as described by Bader et al. (1980), were cured at 100°C which can be considered a low temperature in comparison with the temperature required by modern carbon/epoxy prepregs ($\sim 180^\circ\text{C}$). An estimation of the error introduced by neglecting residual stresses has been carried out here using the methodology described by Nairn (2000) for the approximation of the variation of G with the curing temperature. The maximum relative error expected was about 8% which is low in comparison with the dispersion in Parvizi et al. (1978) results.

Fig. 11 shows the comparison between the experimental results and the theoretical model developed here for the critical strain ε_{yy}^c originating a crack as a function of the inner-ply thickness. Note that the colours of the points representing the experimental results correspond to the colours of the different curves for several values

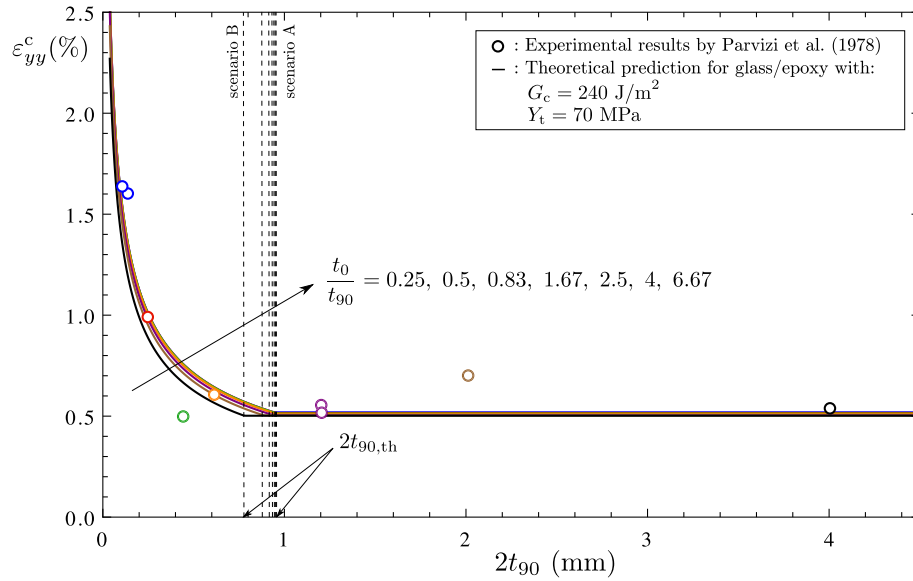


Fig. 11. Experimental values of ε_{yy}^c for various ply thickness in glass fibre/epoxy cross-ply laminates used by Parvizi et al. (1978) and the present theoretical curves predicted for several values of t_0/t_{90} . The arrow shows the direction of increasing values of t_0/t_{90} in the list.

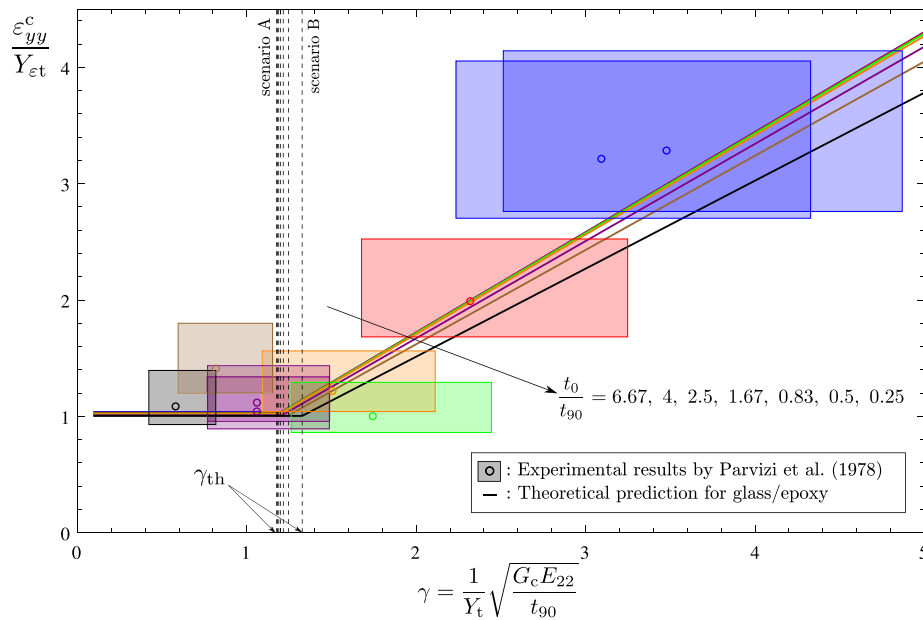


Fig. 12. Experimental values of ε_{yy}^c normalised by the unidirectional ultimate UD transverse strain $Y_{\varepsilon t}$ as a function of inner-ply thickness in glass/epoxy cross-ply laminates from Parvizi et al. (1978) and the theoretical curves predicted for several values of t_0/t_{90} . The arrow shows the direction of decreasing values of t_0/t_{90} in the list.

of t_0/t_{90} . A satisfactory agreement between the experiments and theoretical predictions is remarkably evident from the results presented in Fig. 11.

It is interesting to compare these experimental results with the theoretical predictions as a function of γ where the theoretical predictions have a very simple expression based on two-straight-lines law for geometrically similar laminates. In order to plot the critical strain as a function of γ , its definition (17) is used but, as explained above, a high level of uncertainty exists concerning the values of the strength and fracture properties given by Parvizi et al. (1978). As a consequence, this uncertainty comes into the representation of the experimental results when they are expressed as a function of γ since it depends on Y_t and G_c .

Fig. 12 shows experimental results with the rectangles of uncertainty associated to the operation of expressing the original experimental results in a dimensionless manner. The corners of the rectangles are calculated using the extreme values for Y_t and G_c . First, agreement between the experiments and theoretical model is satisfactory and supports that the simple expression of two straight lines approximates well the experiments. Second, the figure shows that these results, plotted as a function of the structural parameter γ , are more sensitive to the inaccuracy in G_c and Y_t for small values of the inner-ply thickness. In these cases, the rectangles of uncertainty are considerably larger.

The following two particular observations of agreement between the experimental results and theoretical prediction

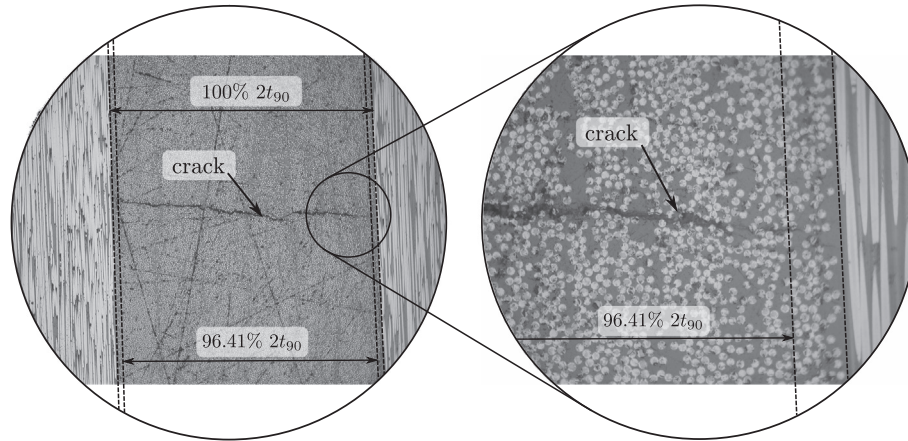


Fig. 13. Example of a transverse crack arrested around the lower bound of the 96.4% on the inner-ply thickness as predicted. Microscope observations from the set of experiments carried out for Blázquez et al., 2009.

should be pointed out. First the threshold length of the inner-ply thickness for which the failure behaviour changes abruptly is very well predicted. Second, the behaviour according to scenario B is also well approximated as can be seen in Figs. 11 and 12.

7.2. Experimental evidences for the arrest length of the cracks

An arrest length of the crack very close to the inner-ply thickness has been predicted by the present theoretical model. In general, all the observed cracks have to satisfy

$$\hat{a}_a \geq \Delta \hat{a}_{\min, g}. \quad (42)$$

From the values of \hat{G} computed numerically by Blázquez et al. (2009) and then obtaining the minimum of an approximation of the function g , it is possible to find the value of $\Delta \hat{a}_{\min, g}$, which depends only on the elastic properties of the plies and on the ply thickness ratio. The value obtained for carbon/epoxy and $t_0/t_{90} = 1$ is

$$\hat{a}_a \geq 0.9641. \quad (43)$$

For usual inner-ply thicknesses and composites, the distance between the arrested crack tip and the interface could be of the same order of magnitude as that of the micromechanical length of reference: the fibre diameter. As a consequence, a more accurate estimation of the crack length than its lower bound cannot be expected. The reason is that the hypothesis of the homogeneous material for the inner-ply is not plausible to predict phenomena associated to this small scale. It would be necessary to take into account the heterogeneous microstructure of the inner ply, which is out of the scope of this work.

Fig. 13 shows a micrograph of a transverse crack in the inner-ply of a laminate tested by París et al. (2010b). This microscopic observation shows that the crack is arrested after approximately reaching the lower bound predicted by the theoretical model developed. A lot of micrographs of cracks in cross-ply laminates can be found in the literature. The majority of those found by the authors show a length of crack very close to the inner-ply thickness. This fact has been described, observed and justified by other approaches arriving to similar results, see e.g. París et al. (2010a,b).

8. Concluding remarks

A novel theoretical model has been developed to predict the first transverse crack onset in cross-ply laminates. Actually, this also refers to the sufficiently distant transverse crack initiations which appear almost simultaneously with the first one. Although

the description has focused on the application to composites, this model can also be applied to other material systems composed by layers with dissimilar elastic properties.

The present model is limited by the validity of the two main hypotheses adopted: Leguillon's hypothesis, basis of the coupled criterion, and the assumption of generalised plane strain before and after the crack onset. This combination has allowed a semianalytical expression for the prediction of the critical strain that originates the first crack to be obtained. This expression depends on the computational results only through a scalar value, which is a function of the elastic properties of the lamina and the geometric parameter t_0/t_{90} .

The theoretical model developed can also predict the final length of the crack achieved after the onset, an arrest length of the crack very close to the inner-ply thickness. This fact is coherent with the experimental results found in the bibliography. A lower bound is also predicted for the arrest length of the crack. This bound is a very accurate estimation of the final crack length because it is very close to the interface (96.4% for carbon/epoxy and 99.86% for glass/epoxy) and a more accurate approximation is not plausible due to the non validity of homogeneous material hypothesis for these distances. Surprisingly, this lower bound value has been demonstrated, in accordance with the model described, to be independent of the strength and fracture properties Y_t and G_c .

A size effect is predicted for the critical strain originating a crack as a function of the inner-ply thickness t_{90} . Apparent strength of the inner ply is predicted to increase for thinner plies. This size effect is a direct consequence of the energy criterion. A simple expression of this size effect for geometrically similar laminates has been obtained as a function of the structural parameter γ , brittleness number. The resulting expression is composed by a constant and a linear function of γ depending on a scalar value $g(\Delta \hat{a}_{\min, g})$.

Experimental evidence from Parvizi et al. (1978) is reanalysed in view of the present theoretical developments. The present theoretical results for the size effect have been confirmed by these experimental results. This theoretical model agrees with the observed threshold value of the inner-ply thickness where the critical strain changes abruptly its behaviour and the predictions for both scenarios with a good accuracy.

A physical interpretation of the size effect has been introduced. It is based on the different geometric dimensions associated to dissipation and release of energy at the crack onset. The energy released by the crack onset is taken from the potential energy stored in a volume. In contrast, the energy is dissipated along a

surface. This difference of dimensions causes that a higher strain is necessary to allow energetically a crack onset to appear when the inner ply is thinner.

As also described by París et al. (2010a) and confirmed experimentally by París et al. (2010b), the study of crack onset is only the first step in the description of this failure mechanism. The next step corresponds to a further increase of load and the appearing of a debond along the interface between both plies. The present approach applied to the problem of the crack initiation can be used to develop a similar theoretical model and to predict the critical parameters for this failure as well. A similar problem has been studied profusely in the last decade, see e.g. Martin et al. (2008) and Leguillon et al. (2013).

Thanks to the simplicity of the model proposed, several applications with practical interest can be developed. For instance, an indirect procedure to measure the transverse fracture toughness, through tests with cross-ply specimens, has been proposed using the present theoretical model by García (2014).

Acknowledgements

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